Hadronic vacuum polarization contribution to muon *g*-2 using staggered fermions

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Project X

with
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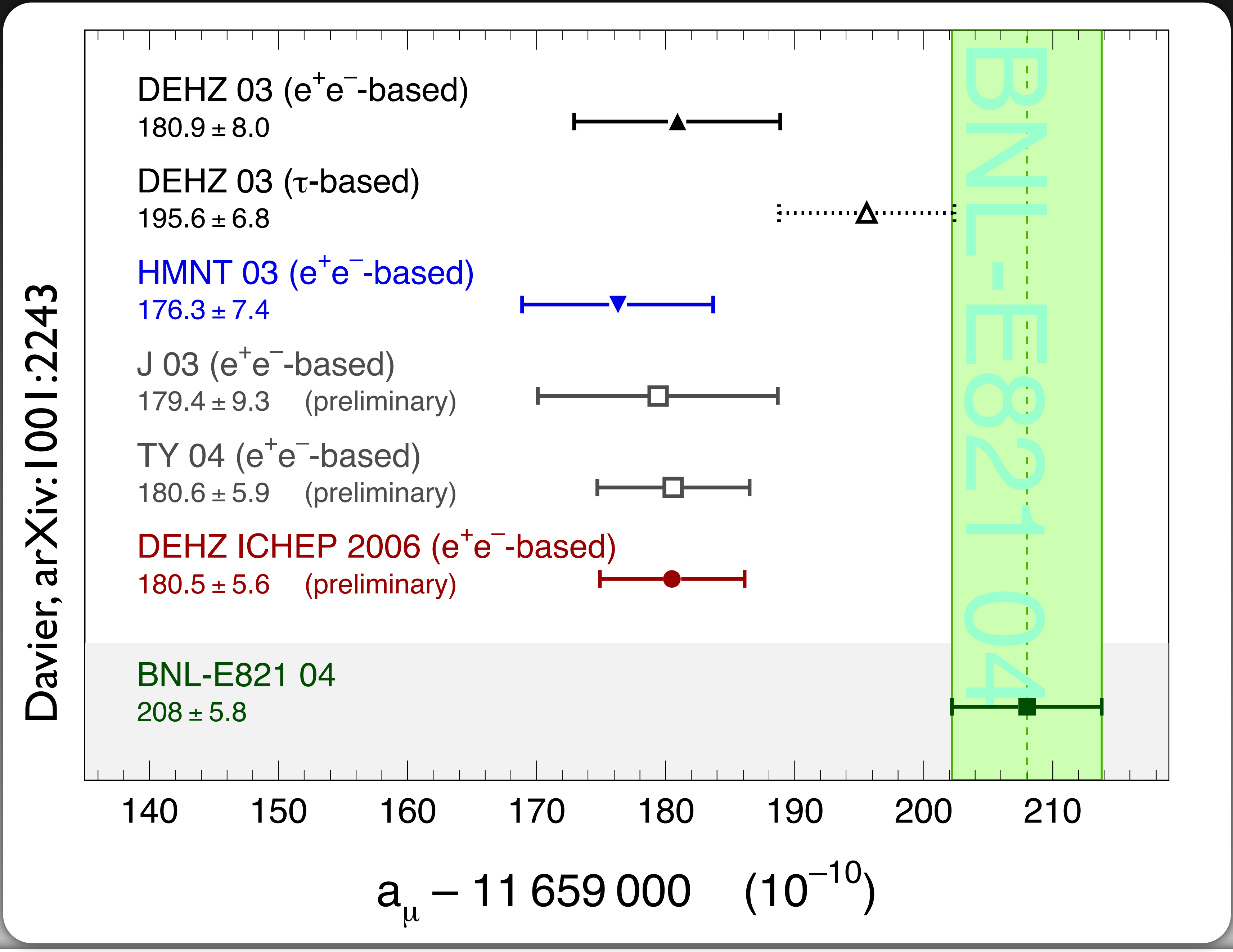
"Theory" vs. Experiment



Deviation ~ 3-3.8 sigma

Fermilab experiment will start in 2016, to reduce error from 0.5 to 0.1 ppm

$$a_\mu^{\rm exp} = \left(\frac{g_\mu - 2}{2}\right)^{\rm exp} = 11\,659\,208(6) \times 10^{-10}$$

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Outline

Muon $g-2$ and current theory for HLO

HLO: $O(\alpha^2)$ Contribution—Vacuum Polarization

$(g-2)^{\mathrm{LHO}}$ from first principles:
Lattice Gauge Theory

Lattice results for vacuum polarization

Preliminary fits for $g-2$
VMD vs. Padé Approximants

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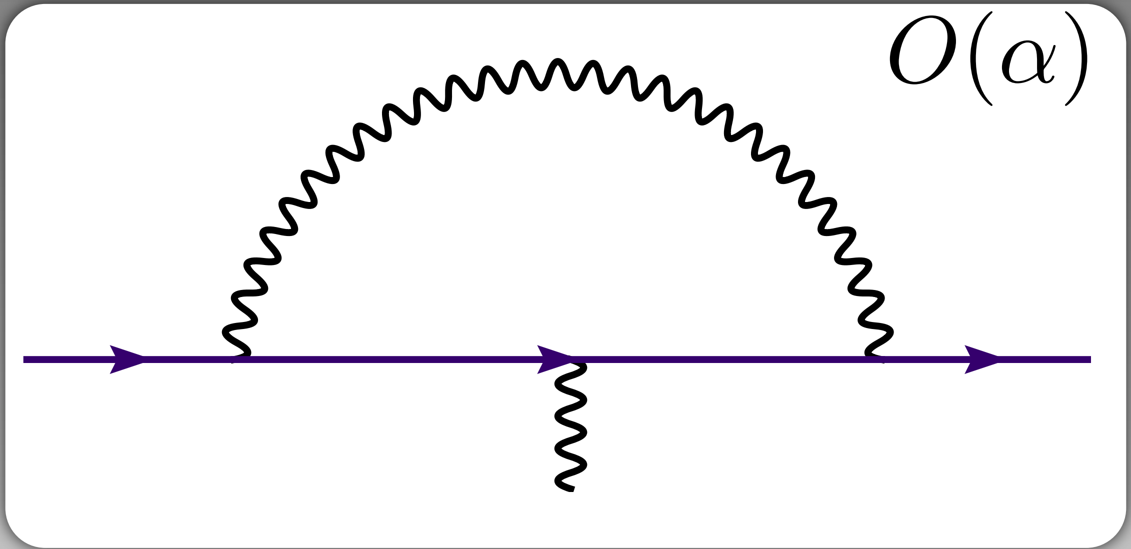
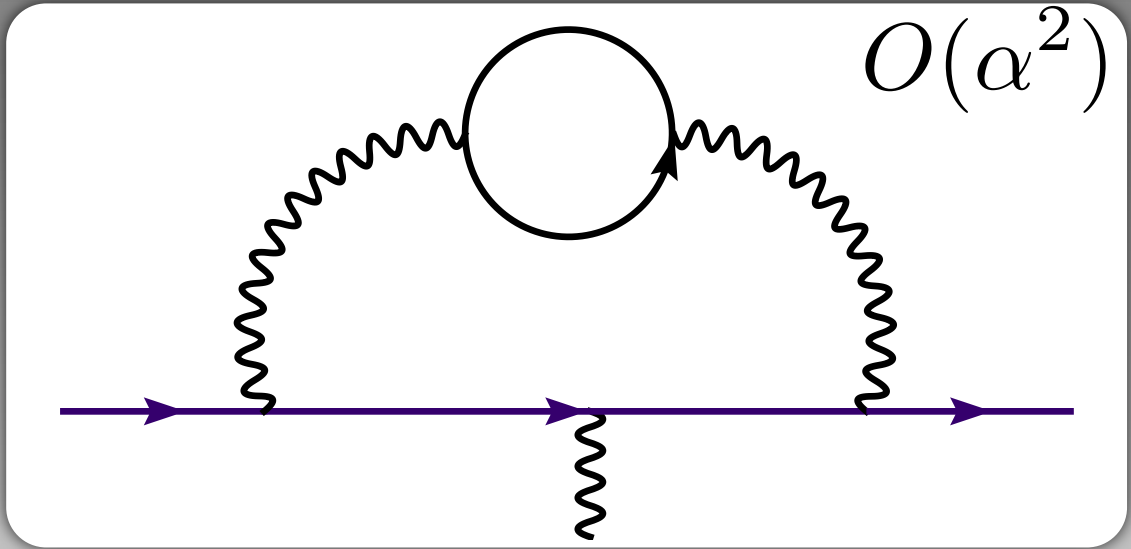
Muon g-2

$$\Gamma^\mu = \gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}}{2m_\mu} F_2(q^2)$$

$$a_\mu = \left(\frac{g_\mu - 2}{2}\right) = F_2(q^2 = 0)$$

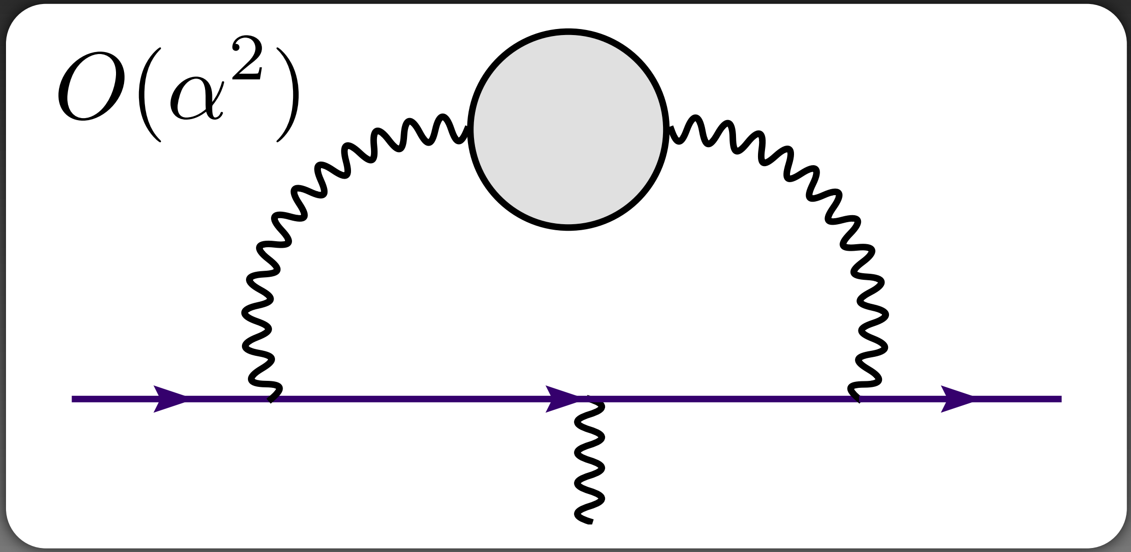




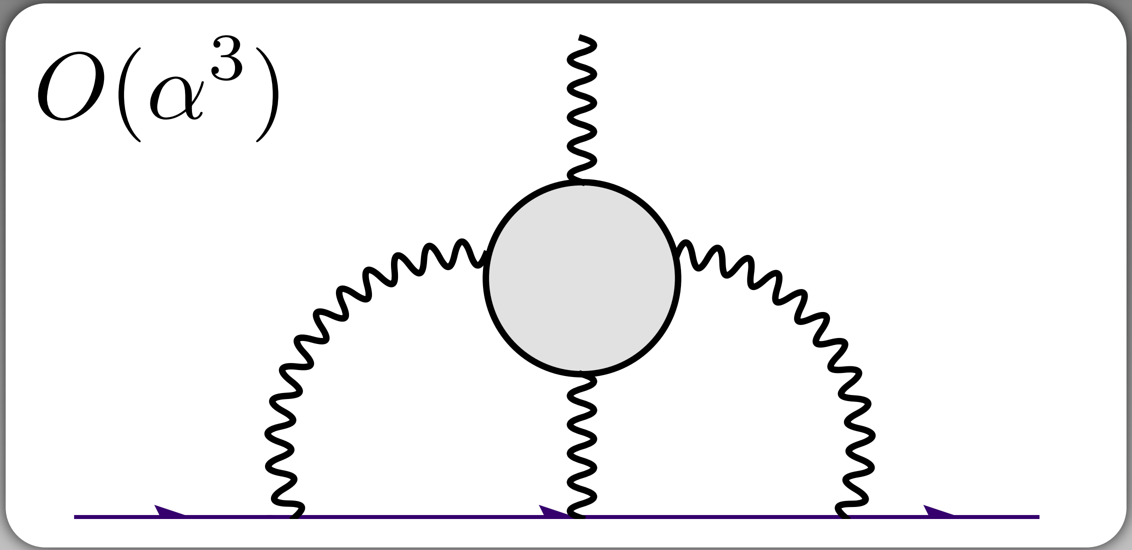
+ other non-QCD terms...

Hadronic contributions

Vacuum Polarization



Light by light



I'll focus on the *leading* hadronic contribution, the vacuum polarization

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Leading Hadronic Contribution

The $O(\alpha^2)$ hadronic contribution, a_μ^{HLO}, cannot be calculated in perturbation theory.

Via the Optical theorem, one can evaluate it using $\sigma(e^+e^- \to \text{hadrons})$.

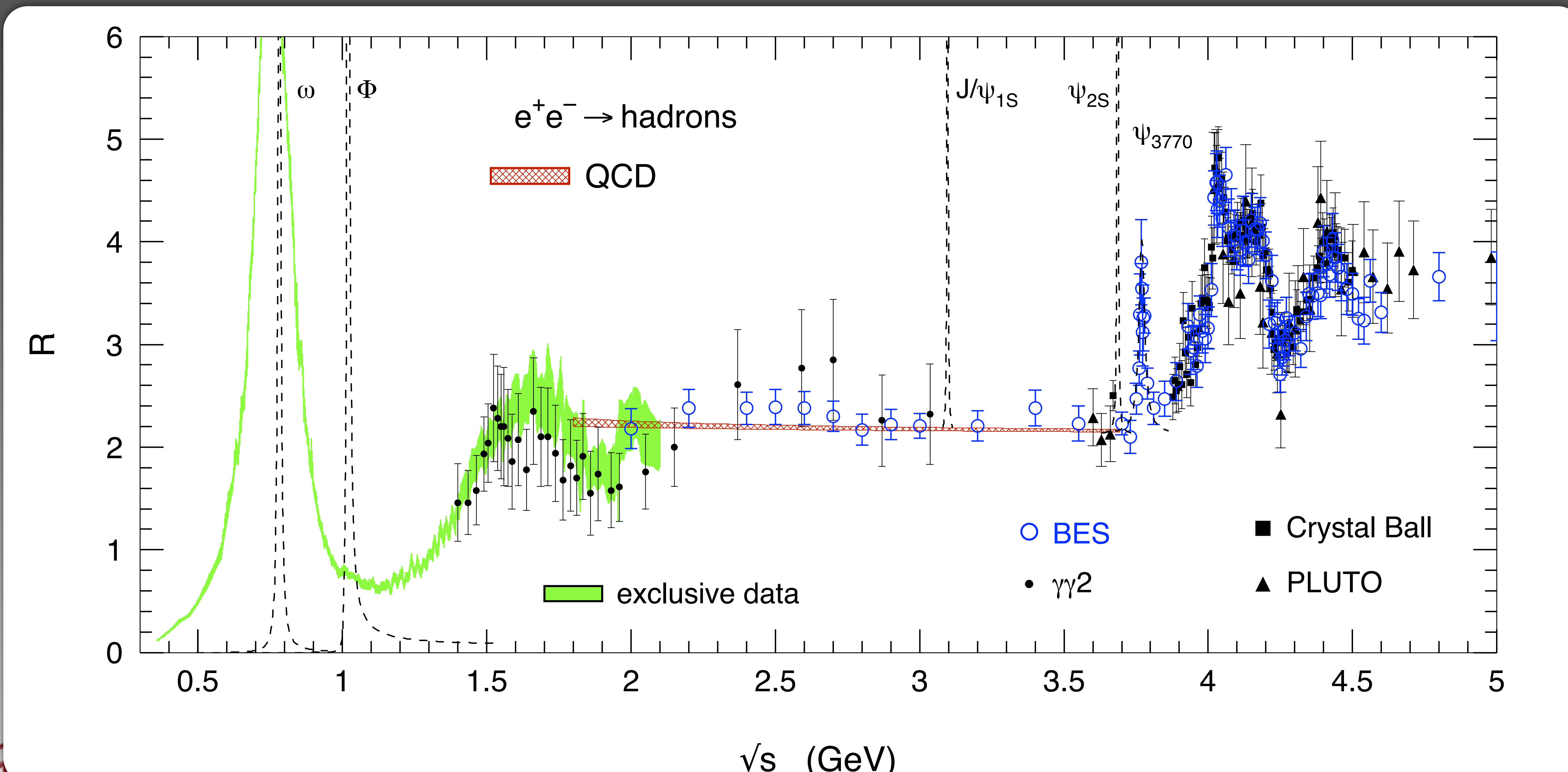
$$a_\mu^{\mathrm{HLO}} = \frac{\alpha^2}{3\pi^2}\int_{4m_\pi^2}^{\infty}\frac{ds}{s}K(s)R(s)$$

The kernel, *K(s)*, is known, and *R(s)* can be measured experimentally

Not a theoretical problem since 1961!

$$R(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

The precision of the Standard Model prediction is limited by the experimental measurement of *R(s)*.



(from Davier et al, hep-ph/0208177)

Lattice QCD

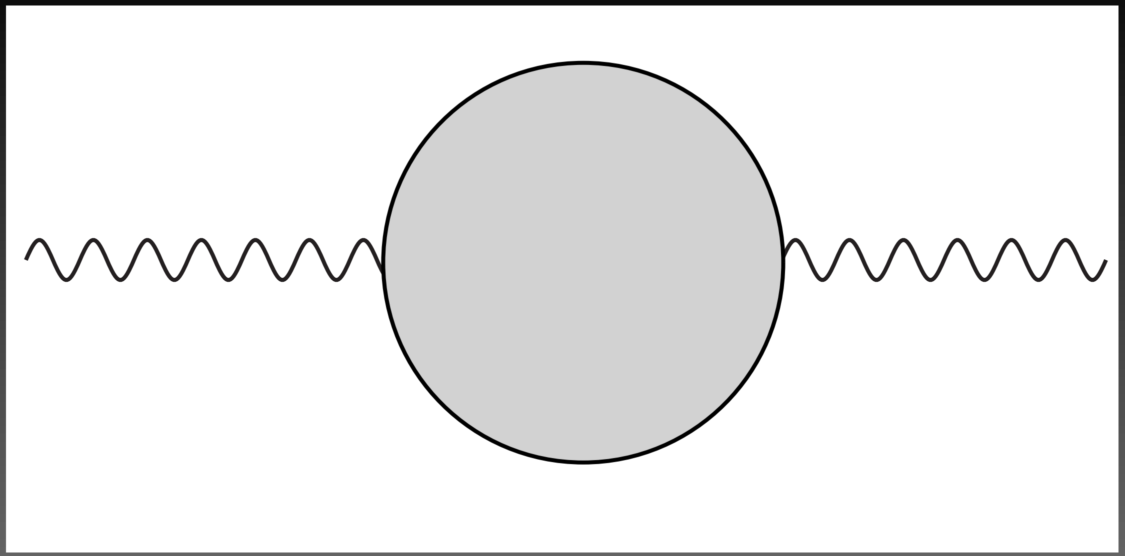
Use staggered quarks (MILC collaboration):

Large volumes
Small quark masses
High statistics

Asqtad staggered (reduced lattice spacing errors)
3 lattice spacings (that we use)
pion masses as low as 180 MeV

Future: HISQ quarks with nearly physical pion masses

Leading Hadronic Contribution



We can extract the $O(\alpha^2)$ hadronic contribution to a_μ from the vacuum polarization using the Euclidean space expression (Blum, 2003)

$$a_\mu^{(2)\mathrm{had,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dK^2 f(K^2)\left[\Pi(K^2) - \Pi(0)\right]$$

$f(K^2)$ diverges as $K^2 \to 0$ ⇒ dominated by low momentum region

⇒Need large lattices to simulate these low momenta accurately

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Lattice Calculation of $\Pi^{\mu\nu}(q)$

Calculate the vacuum polarization using the conserved current

$$\Pi^{\mu\nu}(q) = \int d^4x e^{iq\cdot(x-y)} \langle J^\mu(x) J^\nu(y) \rangle = (q^2 g^{\mu\nu} - q^\mu q^\nu)\Pi(q^2)$$

In the continuum we have the conserved (local) EM current:

$$J^\mu = \overline{\psi}\gamma^\mu\psi$$

While on the lattice it is a point-split current:

$$J_{\mu\ x} = \frac{1}{2}\left[\overline{\psi}_{x+a\hat{\mu}} U^\dagger_{\mu x}(1+\gamma_\mu)\psi_x - \overline{\psi}_x U_{\mu x}(1-\gamma_\mu)\psi_{x+a\hat{\mu}}\right]$$

Satisfies: $\frac{1}{a}\sum_\mu \left[J_{\mu\ x} - J_{\mu\ x-a\hat{\mu}}\right] = 0$

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Lattice Calculation of $\Pi^{\mu\nu}(q)$

On the lattice this satisfies a discrete Ward Identity, so we modify the expressions above

$$Q_\mu \to \hat{Q}_\mu = \frac{2}{a}\sin\left(\frac{aQ_\mu}{2}\right)$$

The finite volume restricts the momenta:

$$Q_\mu = \frac{2\pi n_\mu}{aL_\mu}$$

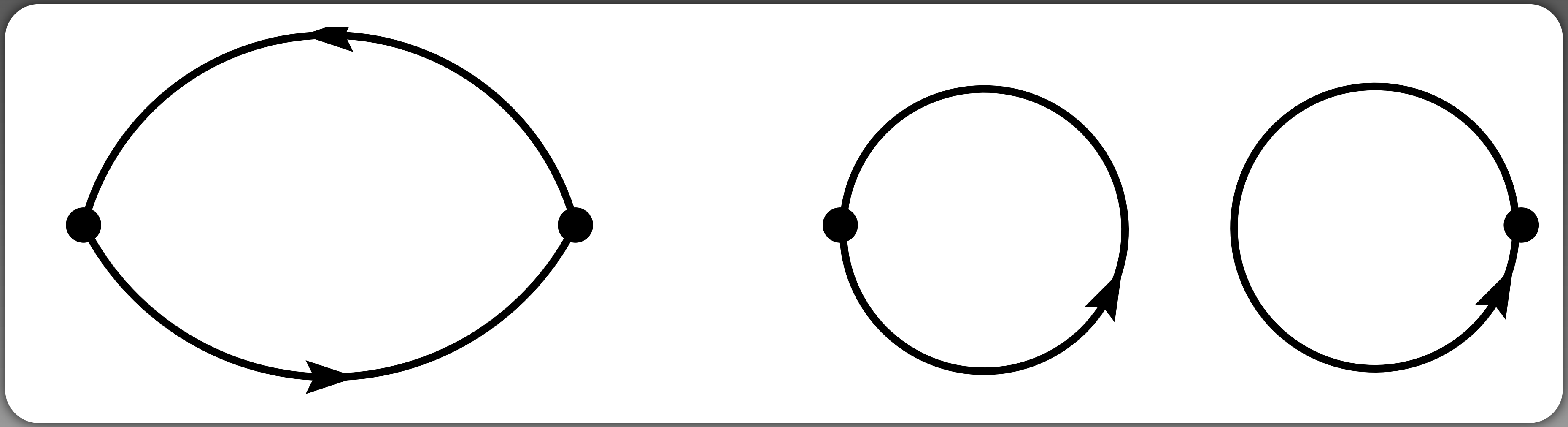
so

$$\Pi^{\mu\nu}(Q) = (\hat{Q}_\mu \hat{Q}_\nu - \hat{Q}^2 \delta_{\mu\nu})\Pi(Q^2)$$

This provides a strong check on the simulation!

Lattice Calculation of $\Pi^{\mu\nu}(q)$

To perform lattice calculation:
Wick contract the quark fields in the current, giving two types of contractions:



We neglect second contraction *for now* (suppressed, also very noisy)

Hard to fit low-momentum region — Also most important part

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(No lattice talk is complete without a table of numbers in unphysical units only the speaker understands)

2+1-flavor MILC "Asqtad" staggered configurations

a (fm)	am_l	am_s	β	size	$m_\pi L$	# lats. avail.
$\approx 0.09^*$	0.0124	0.031	7.11	$28^3 \times 96$	5.78	531
$\approx 0.09^*$	0.0062	0.031	7.09	$28^3 \times 96$	4.14	591
≈ 0.09	0.00465	0.031	7.085	$32^3 \times 96$	4.10	480
$\approx 0.09^*$	0.0031	0.031	7.08	$40^3 \times 96$	4.22	945
$\approx 0.09^*$	0.00155	0.031	7.075	$64^3 \times 96$	4.80	491
$\approx \mathbf{0.09}$	0.0031	0.0031	7.045	$40^3 \times 96$	4.20	440
$\approx 0.06^\dagger$	0.0036	0.018	7.47	$48^3 \times 144$	4.50	751
$\approx 0.06^\dagger$	0.0025	0.018	7.465	$56^3 \times 144$	4.38	768
$\approx 0.06^*$	0.0018	0.018	7.46	$64^3 \times 144$	4.27	826
$\approx \mathbf{0.045}$	0.0028	0.014	7.81	$64^3 \times 192$	4.56	801

Now: 3 lattice spacings, and pion masses as low as ~180 MeV

All (new) simulations were performed using USQCD Lattice resources here at Fermilab (the Ds and J/Psi clusters)

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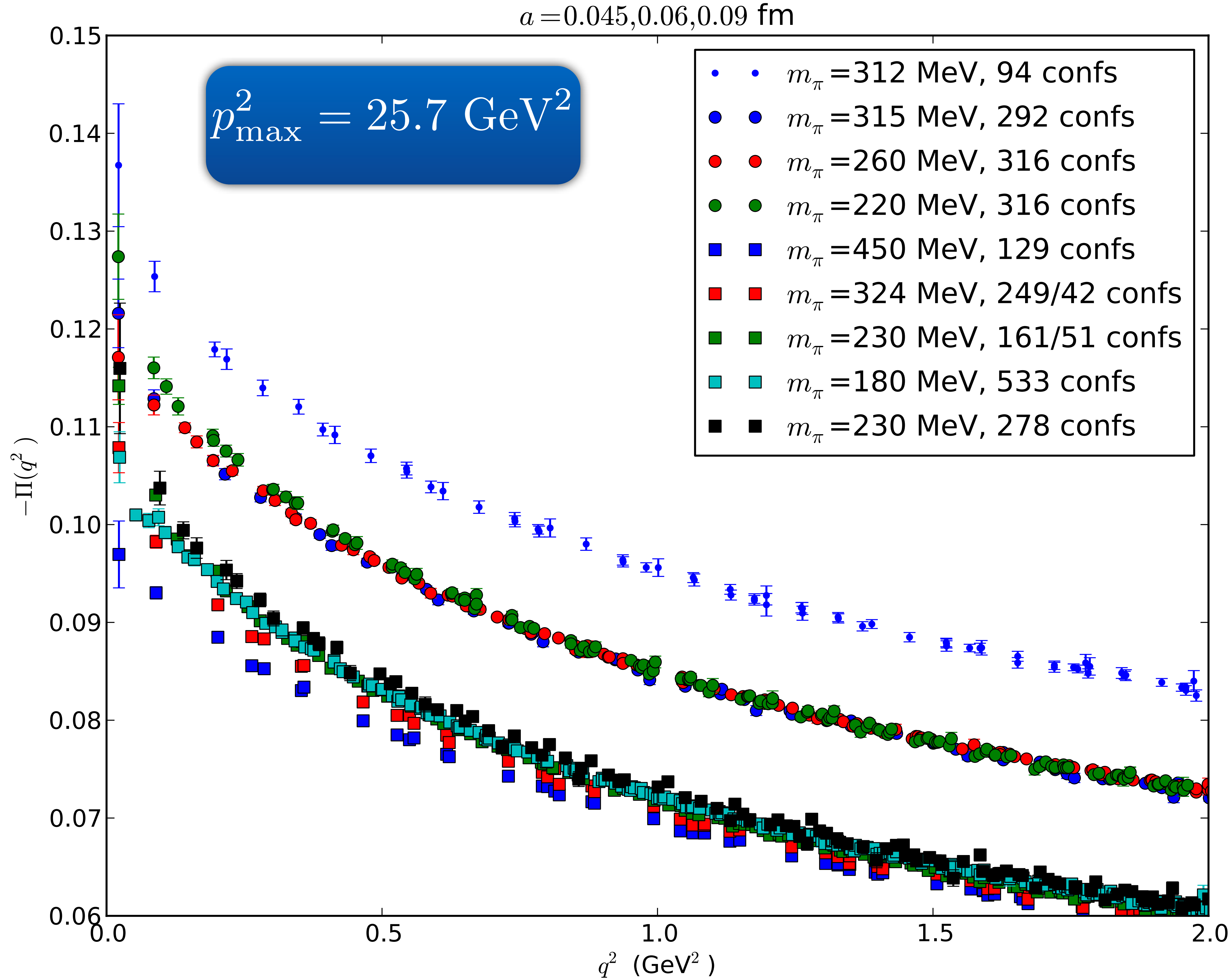
All results (thus far)



Fitting

Can't calculate the vacuum polarization at $q^2=0$ directly

For high momentum, continuum PT works

For low momentum:

Simple polynomials?

No good beyond cubic/quartic order in q^2

Vector Meson Dominance/ChPT

Padé Approximants

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Vector Meson Dominance

$$\Pi(Q^2) = \frac{A}{Q^2 + m_\rho^2} + B$$

A is related to rho decay constant – in principle could determine both this and the rho mass from simulations, thus only *B* is a free parameter

For small pion masses, the two-pion state is the lightest state in this channel – can't measure rho mass (yet)!

Rho decay

On our lightest fine (*a=0.09* fm) lattice, the rho can "decay"

Here, the rho mass cannot be extracted very easily, since

$$am_\rho^{\text{phys}} \approx 0.35 \;(\approx am_\rho^{\text{lat}})$$

And the two smallest non-zero spatial momenta allowed are

$$ap = \frac{2\pi}{L}, \frac{2\sqrt{2}\pi}{L}$$

For a two pion state with rho quantum numbers:

$$aE_{2\pi} = 0.2374, 0.3081 < am_\rho$$

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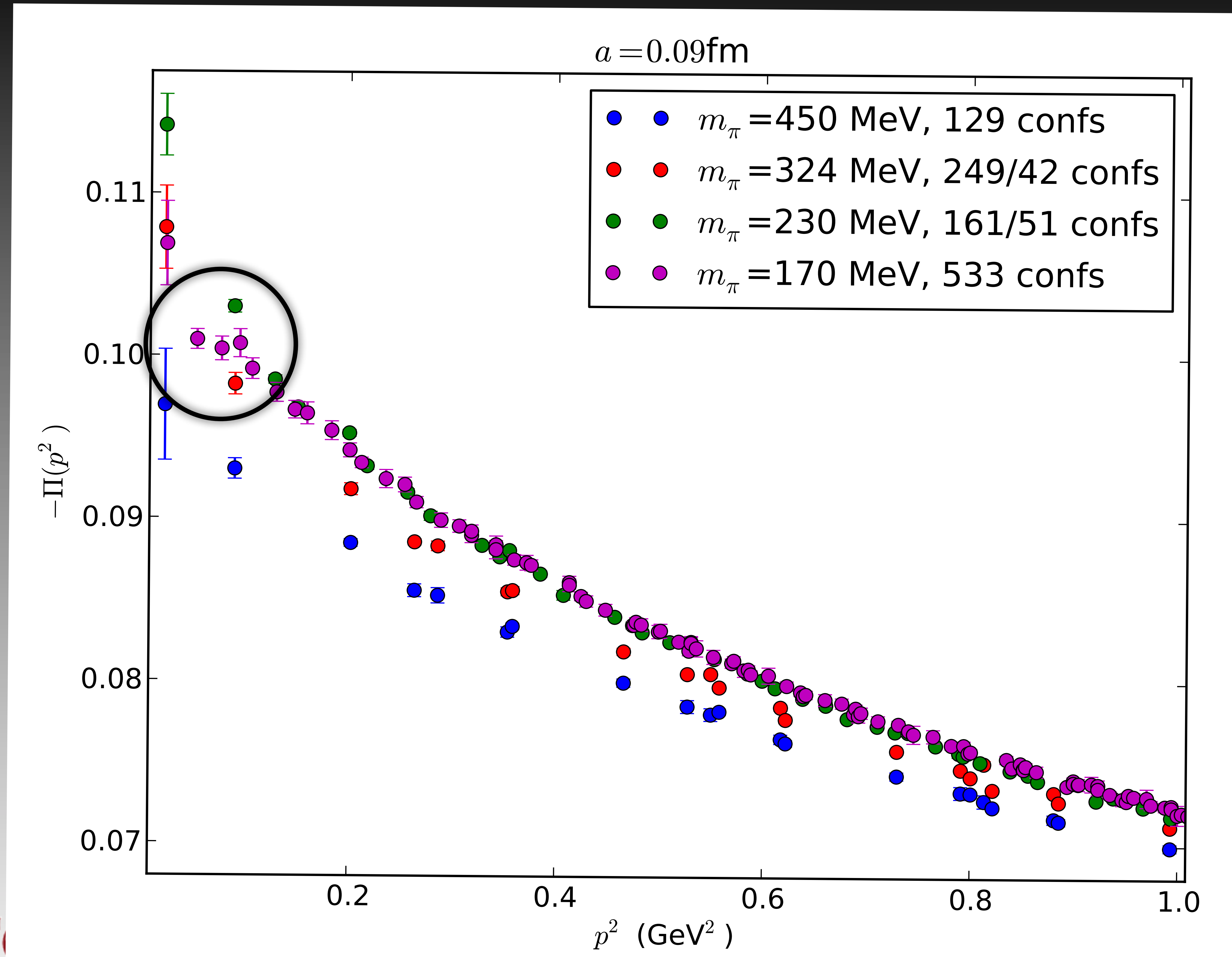
Can be seen to affect the vacuum polarization!



This must be understood theoretically, as it clearly effects the low-energy regime – *cannot get a fully reliable result without it!*

Also tricky to use rho mass as a fixed parameter in fits – without a complete study of mixing with 2-pion state

$$\begin{pmatrix} \langle\rho|\rho\rangle & \langle\rho|2\pi\rangle \\ \langle 2\pi|\rho\rangle & \langle 2\pi|2\pi\rangle \end{pmatrix}$$

Need a full finite-volume analysis to disentangle the rho and 2 pion states

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Padé Approximants

arXiv:1205.3695

$$\frac{\Pi(0)-\Pi(Q^2)}{Q^2} = \int_{4m_\pi^2}^{\infty} dt\, \frac{\rho(t)}{t(t+Q^2)} \equiv \Phi(Q^2)$$

is a *Stieltjes* function, analytic at all points not on the cut $(-\infty, -4m_\pi^2]$

Theorem:

Given P points $(Q_i^2, \Phi(Q_i^2))$, a sequence of Padé Approximants can be constructed which converge to $\Phi(Q^2)$ on any closed, bounded region of the complex plane excluding the cut, in the limit $P \to \infty$. (Baker 1969, Barnsley 1973)

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Padé Approximants

This can be written as

$$\Pi(Q^2) = \Pi(0) - Q^2 \left(a_0 + \sum_{n=1}^{\lfloor P/2 \rfloor} \frac{a_n}{b_n + Q^2} \right)$$

$$a_{n>0} > 0, \;\; b_{\lfloor P/2 \rfloor} > \cdots b_1 > 4m_\pi^2$$

if P is even: $a_0 = 0$

For different values of *P*, we fit to different Padé's

P	2	3	4	5
Padé	[0,1]	[1,1]	[1,2]	[2,2]

For comparison, we will cut off the integral for the g-2 at (1 GeV)^2

Note that VMD is a [0,1] Padé, but with its pole fixed to be the vector mass, and as such is not a valid Padé for our purposes!

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Test on fine MILC lattices (pion mass = 480 MeV)

		correlated interval $0 < Q^2 \leq 0.6$ GeV^2		uncorrelated interval $0 < Q^2 \leq 1$ GeV^2	
PA	# parameters	χ^2/dof	$10^{10} a_\mu^{\mathrm{HLO},Q^2\leq 1}$	χ^2/dof	$10^{10} a_\mu^{\mathrm{HLO},Q^2\leq 1}$
VMD	2	5.86/3*	363(7)	4.37/18	413(8)
[0, 1]	3	11.4/8	338(6)	3.58/17	373(37)
[1, 1]	4	7.49/7	350(8)	3.36/16	424(116)
[1, 2]	5	7.49/6	350(8)	3.35/15	443(293)
[2, 2]	6	7.49/5	350(7)	3.35/14	445(432)

* interval $0 < Q^2 \leq 0.35$ GeV^2

Correlated Padé's are stable – better with more parameters
Higher poles ill-determined (does not affect *g*-2)

Consistent unless one compares uncorrelated VMD with the correlated fits

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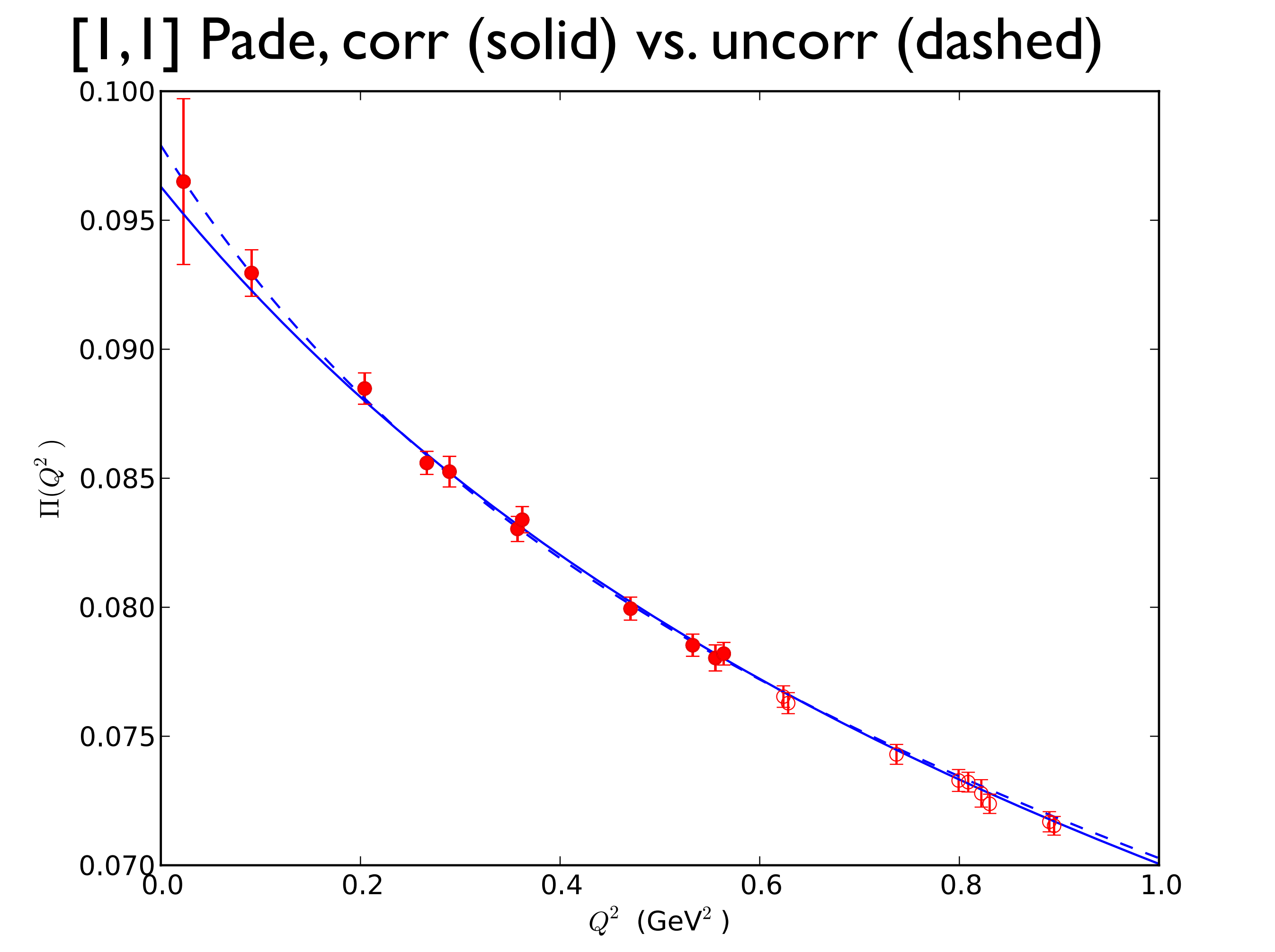
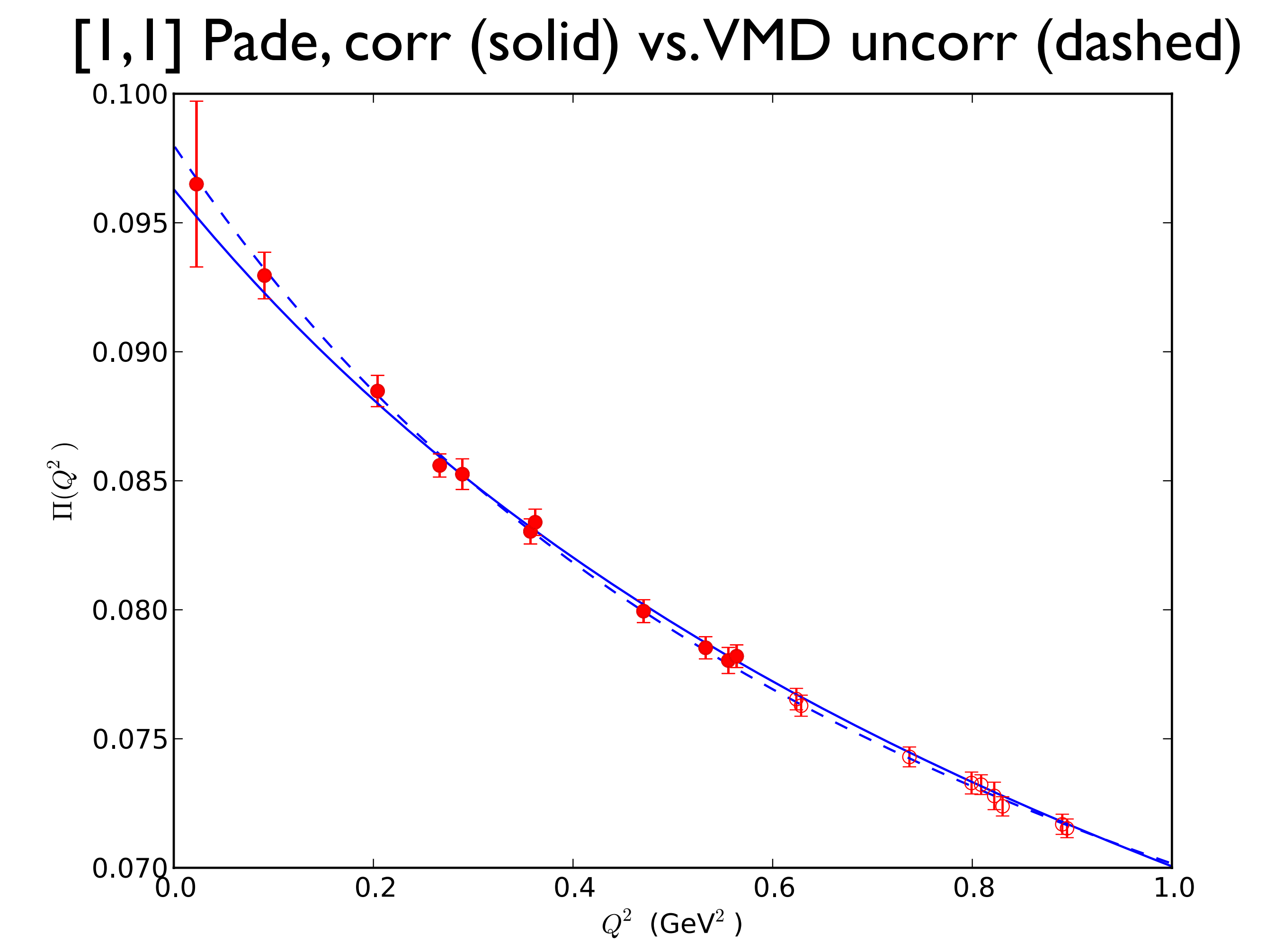




Superfine results:

$a = 0.06$ fm $\quad m_\pi = 220$ MeV

[1,1] Padé (corr):

$$a_\mu^{\mathrm{HLO},\mathrm{Q}^2\leq 1} = 572(41) \times 10^{10}$$

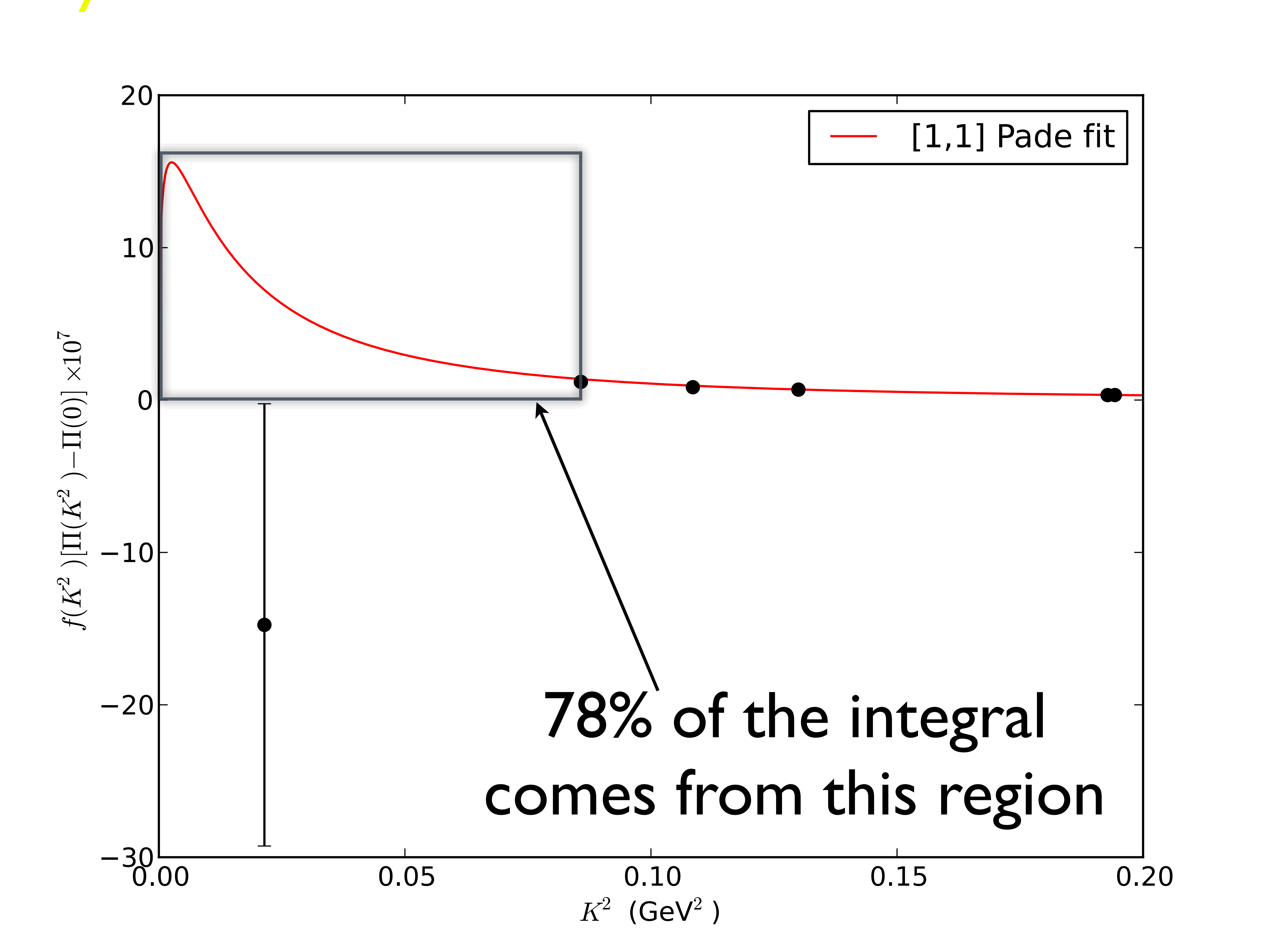
VMD (uncorr):

$$a_\mu^{\mathrm{HLO},\mathrm{Q}^2\leq 1} = 646(8) \times 10^{10}$$

Correlated fits systematically low
All fits have reasonable chi^2

"By eye" – no way to choose one fit over another

Difficulty



[1,1] Pade fit
78% of the integral comes from this region
$f(K^2)[\Pi(K^2)-\Pi(0)]\times 10^7$
K^2 (GeV^2)
20
10
0
−10
−20
−30
0.00
0.05
0.10
0.15
0.20

Note that fits can be misleading!

Unknown systematics are hidden in VMD fits

- Any fits which use data primarily excluding low momentum region should be met with caution!

- 17% discrepancy between VMD & Padé fits

Primary Problems:

- Low momentum (Large volumes/TBC's)
- Statistics (AMA)
- Disconnected contributions (definitely essential for ~5% unc)
- Light quark masses (soon not a problem)

Chiral Extrapolation?

Soon to be irrelevant – HISQ ensembles with near-physical pion mass

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17% discrepancy between VMD & Padé fits

Primary Problems:

Low momentum (Large volumes/TBC's)

Statistics (AMA)

Disconnected contributions (definitely essential for ~5% unc)

Light quark masses (soon not a problem)

Non-trivial problem!

Chiral Extrapolation?

Soon to be irrelevant – HISQ ensembles with near-physical pion mass

Conclusions

Full results still yet to come, analysis complicated by light masses...

Immediate future:

- Better statistics using all-mode averaging (in progress)
- Thus improved fits (able to get higher Padé poles?)
- Begin simulations on HISQ ensembles with nearly physical pion mass

Additionally:

- Need to fill in low momentum region (twisted boundary conditions)

Longer term:

- Include disconnected diagrams

Stay tuned!